AD NUMBER:	AD0838794

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies only Administrative/Operational Use; 01 MAR 1968. Other requests shall be referred to Air Force Eastern Test Range, Patrick AFB FL.

AUTHORITY

SMTC, USAF ltr dtd 22 JUL 1977

ETR-TR-68-10

MINIMUM VARIANCE β ESTIMATION WITH DYNAMIC CONSTRAINTS

John W. McRary, Ph. D. and Lawrence Nicola,
Pan American World Airways, Inc. ASD

March 1968

Each transmittal of this document outside the agencies of the U. S. Government must have approval of Data Processing Division, Directorate of Range Operations, Air Force Eastern Test Range, Patrick AFB, Florida

Data Processing Division
Directorate of Range Operations
Air Force Eastern Test Range (AFSC)
Patrick Air Force Base, Florida

24

BLANK PAGE

NOTICES

This report is available to agencies of the U. S. Government from Defense Documentation Center. Other requesters will secure approval of Data Processing Division, Directorate of Range Operations, Air Force Eastern Test Range, Patrick Air Force Base, Florida 32925. Defense Documentation Center release to the Clearinghouse for Federal Scientific and Technical Information is not authorized.

Distribution of this Document is controlled because of ethical considerations relating to tests.

When U. S. Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

MINIMUM VARIANCE β ESTIMATION WITH DYNAMIC CONSTRAINTS

John W. McCrary, Ph. D. and Lawrence Nicola,
Pan American World Airways, Inc. ASD

FOREWORD

This document was originally published as a working paper and intended for use within the Air Force Eastern Test Range, by its authors, John W. McRary, Ph. D. Senior Systems Engineer, and Lawrence Nicola, Staff Engineer, of Pan American World Airways, Inc. ASD Technical Staff. It was required by the ARIS Reentry Ships Division in support of operations under the Range contract AF 08(606)7500. Project Offices cognizant of the study were the ARIS Reentry Ships Division and the Data Processing Division, Air Force Eastern Test Range.

Review of the working paper indicates the material merits presentation as a technical report. Variation in report format is permitted in the interest of economy, legibility, and expeditious publication.

This technical report has been reviewed and approved.

LOUIS A. MONTALVO
Colonel, USAF
Chief, Instrumentation and Data Processing Division

ABSTRACT

The degree to which the ballistic coefficient (β) can be estimated from reentry metric data can be meaningfully analyzed with a sophisticated computer program which realistically models the problem and performs a proper error analysis of the estimation procedure. Such a program has been developed (at AFETR) and is being used to isolate the primary error sources in the β estimation task. This computer program is additionally used to determine what success in β estimation can be achieved with foreseeable instrumentation accuracies.

Results are included that illustrate the effects on β estimation from the following variations: 1) instrumentation measurement type and accuracy; 2) relative geometry between trajectory and observer; 3) arc length and minimum altitude of tracking; and 4) the magnitude of β .

The central thesis of the analysis presented in this report is that dynamic constraints can be used to advantage in the problem of estimating the trajectory and ballistic coefficient. The least squares, constrained solution takes advantage of the exercise of tracking geometry over a sufficiently long arc to estimate the trajectory parameters and β . For a given set of tracking instrumentation, the enforcement of dynamic constraints throughout the entire trajectory should provide the maximum available information to the estimation process.

(Blank Page)

TABLE OF CONTENTS

BLANK PAGE

I. INTRODUCTION

One of the parameters of interest in current post flight reduction of radar data obtained during a reentry event is the ballistic coefficient history of all objects tracked by the observing instrumentation. The ability to successfully determine the ballistic coefficient β from the measurement data is influenced by the data processing techniques employed. Conversely, a given accuracy in the β estimate can be achieved with minimal instrumentation accuracy if the most efficient data processing is used in reducing the measurement data.

This paper presents the results of an error analysis of β determination capabilities which appraises what can be realistically achieved with foreseeable coherent radar instrumentation, and what sources of error can significantly degrade such estimates. This study was conducted using the recently developed BEAR* (Beta Error Analysis Routine) program, which generates a trajectory and measurement set and performs a minimum variance error analysis of estimated variables, which in this case are the trajectory position and velocity and the ballistic coefficient. This program enforces dynamic constraints in the minimum variance error analysis. The central idea of this analysis is that, for given instrumentation accuracy and a specified tracking geometry, the use of the dynamic constraints should provide maximum information to the β estimation.

Interest in ballistic coefficient estimates is divided into two altitude regimes; the free molecular flow region down to the transition altitude, and the lower altitude regime after continuum flow has been established. Therefore the analysis which follows is divided into an analysis of what can be achieved in each of these regions. In each of these flow regimes the ballistic coefficient can be regarded as constant.

* BEAR is a modified version of the ORAN [1] program.

This study was initiated to examine the degree to which estimation of the ballistic coefficient can be improved by adding a coherent signal processing capability to the advanced Range Instrumentation Ships. This addition would enable the ship to obtain independent doppler data together with range, azimuth and elevation measurements. The experiments for which results and analysis are presented in subsequent sections of this paper were designed assuming the sensor to be a ship at sea.

II. DESCRIPTION OF THE ERROR ANALYSIS PROGRAM

The BEAR program is an IBM 360 program for computing the effects of random and systematic errors on minimum variance orbit determinations. Systematic errors in the instrumentation survey and dynamic model may be considered in the form of either adjusted or unadjusted parameters, with the effects of the latter broken down into individual error sources. The program computes the effects of the unmodeled parameters on both the orbit and recovered parameters, with the orbital effects propagated from epoch to any desired prediction time.

Geopotential parameters plus one drag parameter may be considered. Partial derivatives with respect to these parameters are computed by numerical integration.

The ephemeris of position and velocity is computed analytically by an orbit generator subroutine [2] . In this subroutine the dynamic partial derivatives relating position and velocity at any time to orbit parameters at epoch are also computed analytically, as are the first order secular perturbations due to J_2 and atmospheric drag. Some of the mathematics [3] will be presented here, in a compact form, to describe what is being computed.

First, hypothesize the linearized equations in vector and matrix notation as

$$\underset{n\times 1}{y} = \underset{n\times p}{B}\,\underset{p\times 1}{\gamma} + \underset{n\times u}{U}\,\underset{u\times 1}{\delta} + \underset{n\times 1}{\epsilon} \tag{1}$$

where

y - observation vector from linearization; i.e., the difference between the actual observations and the approximate value

γ - adjusted parameter vector corrections (consists of corrections to the orbital elements, survey, instrumentation biases, etc.)

B - matrix of partial derivatives of the observations (O) and the adjusted parameters

δ - unadjusted or unmodeled parameter vector errors (could consist of geopotential uncertainties, survey errors, etc.)

U - matrix of partial derivatives of the observations and the unadjusted parameters

ϵ - vector of true observation errors. Assume Var $(\epsilon) = \underset{n\times n}{\Sigma}$ a diagonal matrix.

The standard least squares solution for the adjusted parameters, when $\delta = 0$, is

$$\hat{\gamma} = (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} y \tag{2}$$

where

$$\text{Var}(\hat{\gamma}) = (B^T \Sigma^{-1} B)^{-1} . \tag{3}$$

Equation (3) gives the standard result for the accuracy of the adjusted parameters. Conventionally, the square roots of the diagonal elements of $(B^T \Sigma^{-1} B)^{-1}$ are the standard deviations or sigma values for the

adjusted parameters. Now when $\delta \neq 0$, it can be shown that

$$\mathrm{Var}(\hat{\gamma}) = (B^T \Sigma^{-1} B)^{-1} + D W_u D^T \tag{4}$$

where

$$D = (B^T \Sigma^{-1} B)^{-1} B^T \Sigma^{-1} U = \frac{\partial \hat{\gamma}}{\partial \delta} \tag{5}$$

W_u is the variance-covariance matrix of the unadjusted parameters δ, and is taken as a diagonal matrix. The square roots of the diagonal elements of (4) now include a contribution due to the unadjusted effects.

The dynamic constraints are contained in the B matrix, which is computed as the matrix product

$$B = \frac{\partial O_i}{\partial \gamma_o} = \frac{\partial O_i}{\partial X_i} \frac{\partial X_i}{\partial \gamma_o} \tag{6}$$

where the subscripts i and o denote the i^{th} time point and epoch respectively. The dynamic constraints are contained in the second term of equation (6).

For this study, the BEAR computer program was used to provide a minimum variance estimation of the ballistic coefficient simultaneously with the trajectory, where dynamic constraints were enforced. Computer runs were made to examine the effects on β estimation from the following variations:

(1) Instrumentation measurement type and accuracy
(2) Relative geometry between trajectory and observer
(3) Trajectory arc length and minimum altitude of tracking
(4) The magnitude of the ballistic coefficient
(5) Instrumentation measurement biases
(6) Survey uncertainties
(7) Uncertainties in the location of the dynamic center of mass of the earth
(8) The addition of a second sensor.

The results of these studies are discussed in the next section.

III. ERROR ANALYSIS

In the high altitude free molecular flow region, the atmospheric drag force

$$\bar{F}_D = -\tfrac{1}{2}\,\rho\, m\, g\, \frac{V\,\bar{V}}{\beta}\,, \quad \text{with} \tag{7}$$

$$\beta = \frac{W}{C_D A} \quad \frac{\text{lb}}{\text{ft}^2}\,,$$

is quite small [4]. Attempts to measure these minute decelerations at each point in time and thereby determine β on a point-to-point basis translate into unattainable instrumentation accuracy. However, a long arc solution which exploits dynamic constraints and variable geometry can see a cumulative effect which is not detectable with point-to-point observations.

As an object makes the transition from free molecular to continuum flow, the ballistic coefficient increases. For purposes of discussion, it is reasonable to assume that an object which has hypersonic continuum flow β of 500-1000 will have a free molecular flow ballistic coefficient of about 150-200. In all cases presented here no a priori information was assumed for the trajectory, which is estimated simultaneously with the ballistic coefficient.

Figure 1 illustrates the results of a BEAR analysis with the parameter σ_β/β chosen as the figure of merit in the following curves. The true ballistic coefficient for this curve is 160 and the reentry parameter at 400K feet are $\gamma_E = 34^o$ and $V_E = 2 \times 10^4$ ft/sec. A single doppler radar with the indicated measurement capability, located approximately 50 miles downrange of impact, is the observing instrument. These accuracies were chosen to represent reasonable state-of-the-art capability in the future. Moreover, very large

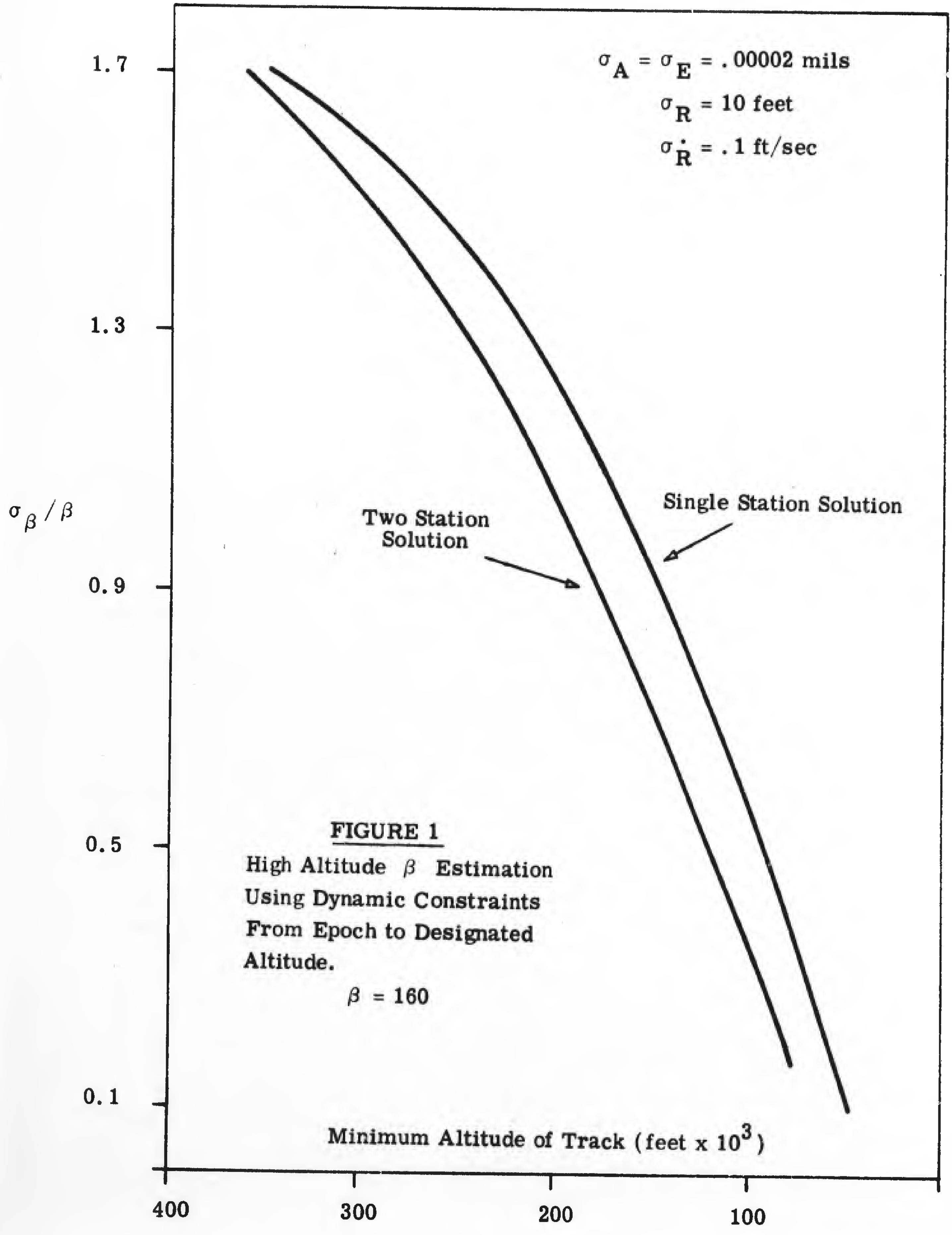

FIGURE 1

High Altitude β Estimation Using Dynamic Constraints From Epoch to Designated Altitude.

$\beta = 160$

values were assumed for the a priori σ_β to simulate a lack of knowledge of the value of β. By this means the estimate is based entirely on measurement information collected during the event. This curve has been extended to lower altitudes because the transition point is dependent on body shape according to the relation $\frac{\lambda}{D} < .01$ so that the lower cutoff point is variable, where λ is an atmospheric mean free path and D is the body diameter. Initial acquisition of the object occurs approximately 6.5 minutes before impact at a range of 10^3 miles. The measurements are summarized in Figure 2.

Under these conditions, the curves show that it is unlikely that β can be resolved to within a factor of two for objects of this class ($\beta_{cont} \sim 500 - 1000$) before the transition flow regime occurs. If another identical radar, located some 50 miles uprange and displaced from the trajectory is available to assist in the discrimination, Figure 1 shows that the situation is only slightly improved in the high altitude region.

The ability to estimate β is not directly a function of the ability to measure position and velocity. Table 1 shows the position and velocity standard deviations at points of interest on the trajectory corresponding to Figure 1. Notice that even though the trajectory is improved by a factor of two, the β estimate improves by approximately 20%. The dynamic constraint filter is greatly enhanced by the doppler measurement of one component of velocity, so the optimum configuration is a sensor located to maximize the doppler component of velocity over the β estimation interval. Table 2 gives a comparison of the ability to estimate β and the trajectory with and without an $\dot{R}$ measurement, for two values of β. Even though the doppler measurement does not greatly affect the trajectory determination, it directly influences the β estimate. In fact, studies have shown that for a given geometry and instrumentation accuracy, σ_β very nearly scales with $\sigma_{\dot{R}}$.

A determination of the ballistic coefficient at lower altitudes in continuum flow regime is of interest primarily for intelligence purposes rather than discrimination information on uncooperative objects.

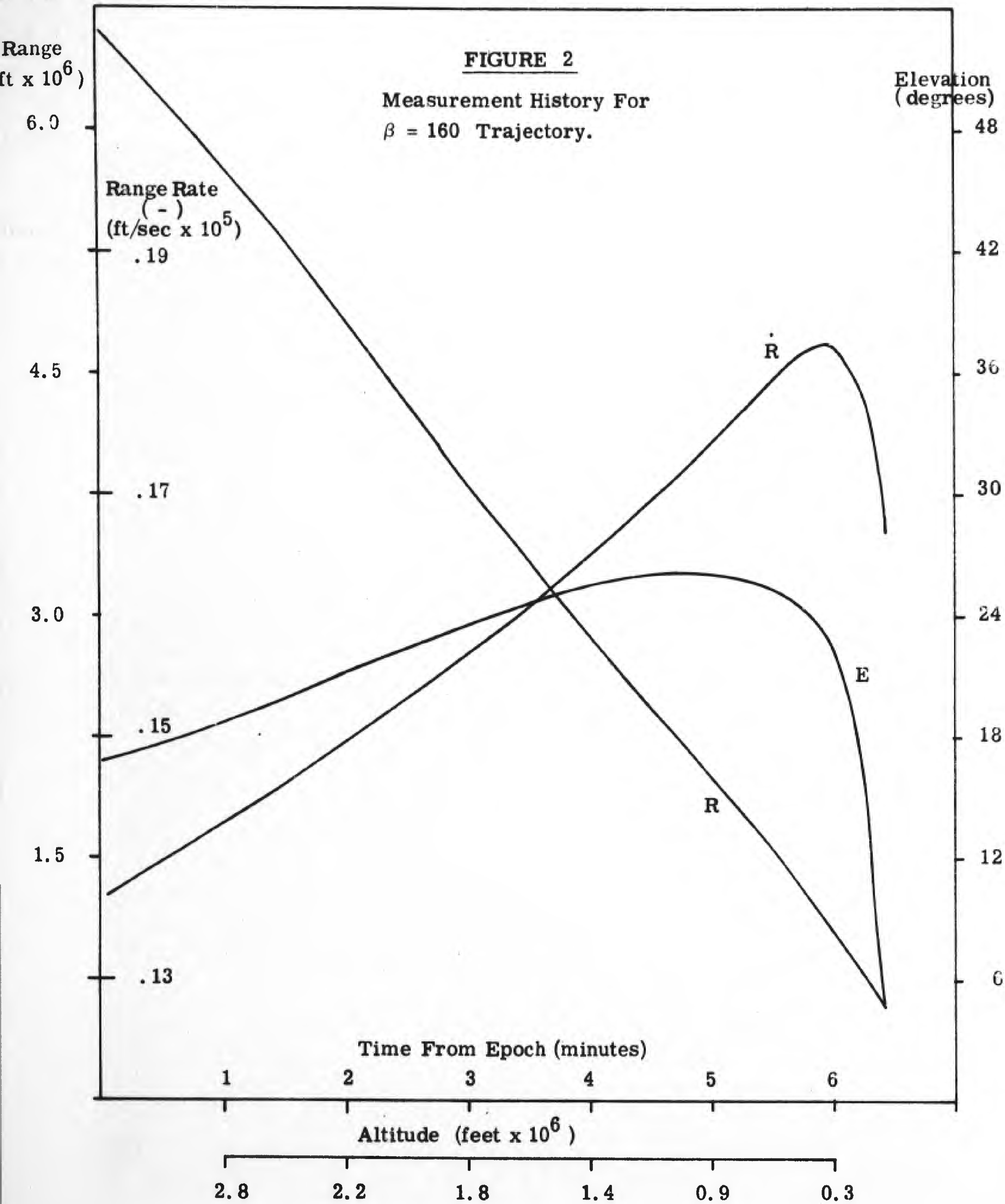
FIGURE 2
Measurement History For
β = 160 Trajectory.
Range
(ft x 10^6)
6.0
4.5
3.0
1.5
Range Rate
(-)
(ft/sec x 10^5)
.19
.17
.15
.13
Elevation
(degrees)
48
42
36
30
24
18
12
6
Ṙ
E
R
Time From Epoch (minutes)
1
2
3
4
5
6
Altitude (feet x 10^6)
2.8
2.2
1.8
1.4
0.9
0.3

TABLE 1

Time From Epoch (min)	Single Ship σ_p (ft)	Single Ship σ_v ($\frac{ft}{sec}$)	Two Ships σ_p	Two Ships σ_v
0	8.67	.03	4.66	.01
1.0	7.25	.03	3.88	.01
2.0	5.71	.03	3.02	.01
3.0	4.29	.03	2.24	.01
4.0	2.99	.03	1.52	.01
5.0	2.05	.03	1.02	.01
6.0 (320 K ft)	2.03	.03	1.09	.01
6.2 (197 K ft)	2.16	.03	1.18	.01
6.35 (cut off measurements) 3.7 K ft.	2.27	.08	1.24	.06

TABLE 2

σ_β / β a priori = 2.0

β	$\sigma_{\dot{R}} = .1$ σ_β/β	σ_v	σ_p	No $\dot{R}$ Data σ_β/β	σ_v	σ_p
160	.12	.03 $\frac{ft}{sec}$	2.0 (ft)	1.9	.06	3.0
1600	.23	.02	1.5	1.97	.06	3.0

The ability to estimate β below 300K feet as a function of acquisition altitude is shown in Figure 3 for two different values of the ballistic coefficient. The power of dynamic constraints is illustrated in Table 3, which compares σ_β / β as determined by the BEAR program with values obtained by ten second filtering.

The ability to determine β is a function of instrumentation location relative to the trajectory plane. Table 4 compares σ_β / β determinations for radar locations 50 miles downrange, crossrange, and at a 45° angle with respect to the trajectory plane.

As a final topic, an example of the effects of unadjusted parameters on β determination will be presented. A single station solution is considered with the standard deviation of the errors shown in Table 5.

If propagated as uncorrelated errors, they contribute to the error in β as shown in Table 6. If these error sources exist in the instrumentation it would be necessary to adjust for these parameters along with β. Table 6 shows the results of an attempt to adjust the errors and the residual effect on σ_β.

Survey errors for a single station need be included only if it is necessary to express the measured trajectory relative to a known coordinate system. For example, this will be the case when it is desired to determine the launch point for intelligence purposes. For β determination it is necessary to estimate the trajectory relative to a single station, hence only the error in the dynamical center of the earth relative to the ship need be considered. An analysis similar to that of Table 6, where the relative uncertainty in the radial distance to the dynamical center was assumed to be 150 feet, has shown that this effect is less than one percent on σ_β / β. Survey errors could also be significant when it is desired to combine the measured trajectory with data from another station. Consider again the two-station solution of Figure 1, where the second radar improves the β determination by approximately twenty percent. If an error of 2500 feet is assumed

TABLE 3

a priori $\sigma_\beta / \beta = 10$

	σ_β/β Dynamic Constraints	σ_β/β 10 Sec. Filter
150 K ft, $\beta = 800$	.55 (Fig. 3)	1.84
250 K ft, $\beta = 800$	.42 (Fig. 3)	10
300 K ft, $\beta = 160$	1.5 (Fig. 1)	10

TABLE 4

$\beta = 800 \text{ lb/ft}^2$

	σ_β/β	σ_p	σ_v
Downrange	.20	1.75	.02
45^o Angle	.32	1.2	.02
Crossrange	.35	1.35	.02

This table again illustrates the power of the $\dot{R}$ measurement in determining β .

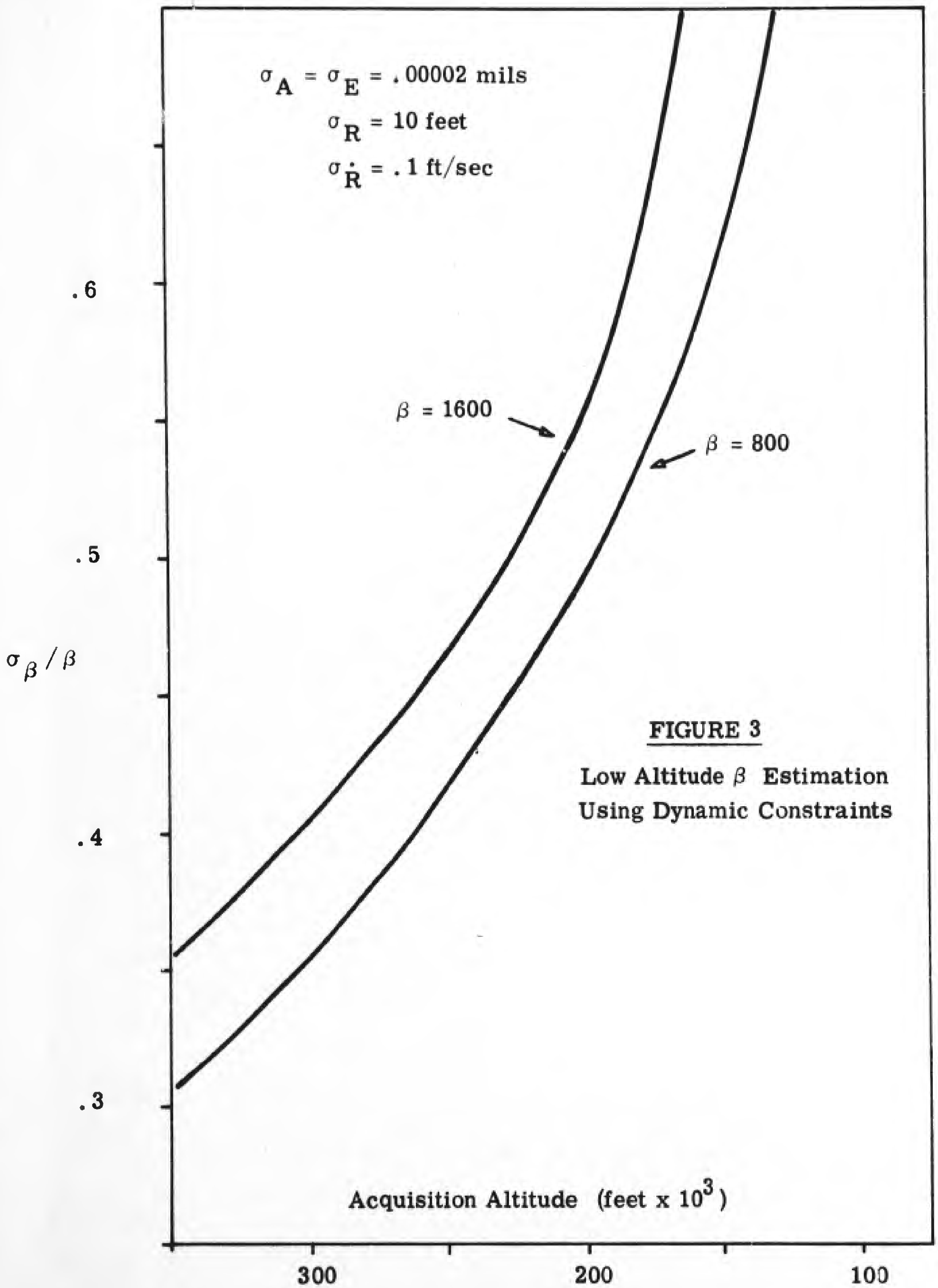

FIGURE 3

Low Altitude β Estimation Using Dynamic Constraints

TABLE 5

		a priori σ	Adjusted σ
1.	R Bias	50 ft	21.3 ft
2.	A Bias	.00066 mil	.00004 mil
3.	E Bias	.00066 mil	.00011 mil
4.	E Refraction	.1 mil	.05 mil
5.	X Survey	2500 ft	2144.0 ft
6.	Y Survey	2500 ft	1175.0 ft

TABLE 6

	σ_β / β No Biases	σ_β / β Unadjusted Biases	σ_β / β Adjusted Biases
Ballistic Coefficient	.208	18.95	.431

Individual Effects of Unmodeled Parameters on σ_β

Unadjusted Parameters	Contributions To σ_β / β
R Bias	.75
A Bias	.37
E Bias	18.6
E Refraction	.4
X Survey	2.9
Y Survey	1.5

along two aces for the uncertainty in the relative location of one of the ships, these bases will completely negate the improvement in β estimation gained by the presence of the second ship.

It is of particular interest to the Aris ship analysis to determine what precision in $\dot{R}$ is required to improve the β determination capability. Table 7 presents a matrix of σ_β / β estimation capability as a function of $\dot{R}$ precision and ship location relative to impact. The present capability with no $\dot{R}$ is also included for comparison. This data shows that little benefit is derived from $\dot{R}$ measurements with $\sigma_{\dot{R}} > .2$ ft/sec , where the ship is located within a 45° cone with respect to the trajectory plane.

TABLE 7

$\beta = 800$ lb/ft^2

σ_β / β a priori = 1.0

$\sigma_{\dot{R}}$	Downrange	45° Angle	Crossrange
2.0	1.0	1.0	1.0
.2	.65	.65	.92
.1	.20	.32	.35
No $\dot{R}$	1.0	1.0	1.0

IV. CONCLUSIONS

1. In both free molecular and continuum flow the use of dynamic constraints appears to have potential for β estimation.

2. For slender bodies where the atmospheric drag force is relatively small, and in the high altitude regions of the atmosphere, the dynamic constraint technique appears to offer significant advantage.

3. For the cases examined, range, azimuth, and elevation primarily determine the trajectory, while the inclusion of an $\dot{R}$ measurement greatly increases the capability to estimate β .

4. Because of the importance of $\dot{R}$ in the determination of β , it is desirable to align the sensor with the trajectory plane.

5. Bias errors in the measurements, especially elevation angle errors in this case, seriously degrade β estimation if they cannot be removed by calibration.

6. Data reduction techniques that can simultaneously adjust for the trajectory, β and instrumentation biases appear to offer the possibility of significant improvement in β estimation.

7. Survey errors for a single sensor are significant only when transforming the measurements from the sensor referenced coordinate system; otherwise they do not influence the β determination.

8. Uncertainties in the location of the dynamical center of the earth has negligible effect on β estimates obtained using dynamic constraints.

9. The addition of a second sensor near the impact point contributes on the order of twenty percent to the β estimation capability. However, relative survey errors can negate this improvement.

REFERENCES

[1] Martin, C. F., "ORAN, A Computer Program for Estimating the Effects of Unmodeled Errors on Minimum Variance Orbital Determinations," PAA Technical Staff Technical Memo No. 79, ETV-TM-68-12, 7 August 1967.

[2] Nicola, L., "ORGEN, The Subroutine of ORAN That Generates the Orbit and Computes the Dynamic Partials," Pan Am Technical Staff Technical Memo No. 81, ETR-TM-68-25, 15 September 1967.

[3] Bush, N., "The Effect of Unmodeled Errors on Orbital Estimation," Pan Am Technical Memo No. 89, Paper to be presented at Fifth Space Congress, Cocoa Beach, Fla., 11 March 1968.

[4] Martin, C. F., and McRary, J. W., "Survey of Reentry Radiation Phenomena," (U), Pan Am Technical Staff Memo No. 31, ET-65-26491, (SECRET), 20 August 1965.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
Pan American World Airways, Inc. ASD
Patrick AFB, Florida 32925

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP
N/A

3. REPORT TITLE
Minimum Variance β Estimation with Dynamic Constraints

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Originally prepared as a working paper for internal use Air Force Eastern Test Range

5. AUTHOR(S) (First name, middle initial, last name)
John W. McRary, Ph. D.
Lawrence Nicola

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1968	21 incl. prefatory	four

8a. CONTRACT OR GRANT NO.
AF 08(606)7500

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
TM 90/ETV-TM-68-108

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
ETR-TR-68-10

10. DISTRIBUTION STATEMENT This report is available to agencies of the U. S. Government from Defense Documentation Center. Other requesters will secure approval of Data Processing Division, Directorate of Range Operations, Air Force Eastern Test Range, Patrick AFB, Fla. DDC release to Clearinghouse for Federal Scientific & Technical Information is not authorized.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Data Processing Division, Directorate of Range Operations, AFETR (AFSC), Patrick AFB, Florida

13. ABSTRACT

The degree to which the ballistic coefficient (β) can be estimated from reentry metric data can be meaningfully analyzed with a sophisticated computer program which realistically models the problem and performs a proper error analysis of the estimation procedure. Such a program has been developed (at AFETR) and is being used to isolate the primary error sources in the β estimation task. This computer program is additionally used to determine what success in β estimation can be achieved with foreseeable instrumentation accuracies.

Results are included that illustrate the effects on β estimation from the following variations: 1) instrumentation measurement type and accuracy; 2) relative geometry between trajectory and observer; 3) arc length and minimum altitude of tracking; and 4) the magnitude of β. The central thesis of the analysis presented in this report is that dynamic constraints can be used to advantage in the problem of estimating the trajectory and ballistic coefficient. The least squares, constrained solution takes advantage of the exercise of tracking geometry over a sufficiently long arc to estimate the trajectory parameters and β. For a given set of tracking instrumentation, the enforcement of dynamic constraints throughout the entire trajectory should provide the maximum available information to the estimation process. ()

DD FORM 1 NOV 65 1473

Unclassified
Security Classification

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ballistic trajectories						
Target recognition						
Trajectory (estimation)						
Reentry vehicle (constraints)						
Missile signature						

Unclassified
Security Classification